II. **Solution by J. SCHEFFER, A. M.. Hagerstown, Md.**

Let ACB be the triangle; choose $BC=a$ for the axis of abscisses, and CA for that of ordinates. Any circumscribed ellipse is of the form $y^2+Axy+Bx^2+Cy+Dx=0$; and since $(a, 0)$ and $(0, b)$ are points of the ellipse, we have $C=-b$, $D=-Ba$, and the above equation reduces to $y^2+Axy+Bx^2-by-Bax=0$.

Transforming it to the center of the ellipse as origin, it reduces to

$$y^2+Axy+Bx^2-\frac{Bb^2-ABab+B^2a}{4B-A^2}=0.$$

The area is$=\pi$ sin$C$. $\dfrac{Bb^2-ABab+B^2a}{4B-A^2}=m$. Developing $\dfrac{\partial m}{\partial A}=0$, and $\dfrac{\partial m}{\partial B}=0$, we find $A=\dfrac{b}{a}$ and $B=\dfrac{b^2}{a^2}$, and thus find the ellipse of minimum area to be, $a^2y^2+abxy+b^2x^2-a^2by-ab^2x=0$. The center is the point $(a/3, b/3)$. The maximum ellipse about the triangle is concentric, and its equation is

$$a^2y^2+abxy+b^2x^2-a^2by-ab^2x+\frac{a^2b^2}{4}=0.$$

Also solved by C. N. Schmall, and V. M. Spunar.

280. **Proposed by C. N. SCHMALL, 89 Columbia Street, New York.**

Find the envelope of the system of spheres

$$\left.\begin{array}{r}(x-a)^2+(y-b)^2+z^2=r^2\\ a^2+b^2=c^2\end{array}\right\}.$$

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; V. M. SPUNAR, Pittsburg, Pa., and the PROPOSER.

Differentiating $(x-a)^2+(y-b)^2+z^2=r^2$ and $a^2+b^2=c^2$ with reference to a as the independent variable, we have $(x-a)+(y-b)\dfrac{\partial b}{\partial a}=0$, and $a+b\dfrac{\partial b}{\partial a}=0$; from the second equation we get $\dfrac{\partial b}{\partial a}=-\dfrac{a}{b}$, and substituting in the first we get $(x-a)-\dfrac{a}{b}(y-b)=0$, whence $b=\dfrac{ay}{x}$ and combining this with $a^2+b^2=c^2$, we get $a^2=\dfrac{c^2x^2}{x^2+y^2}$, $b^2=\dfrac{c^2y^2}{x^2+y^2}$, and substituting this in the first given equation, we have, after some easy reductions:

$$x^2+y^2+z^2-2c\sqrt{(x^2+y^2)}=r^2-c^2.$$

This is, as can easily be seen, the equation of the surface of a ring, the central line of which is a circumference whose radius is$=c$, and the perpendicular section of a circle whose radius is$=\sqrt{(2c^2-r^2)}$.